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GREEK PHILOSOPHERS ON THE DISCIPLINARY VALUE OF MATHEMATICS.

By Florian Cajori.

Two independent methods have been used for the determination of the disciplinary value of mathematics: (1) the method of laboratory measurement, which, though ambitiously pursued, has not yet yielded altogether consistent results;[1] (2) the historical method which recounts the experiences and opinions of the great teachers and philosophers of past ages. It is the latter method that we wish to consider in this paper, particularly in reference to Greek testimony.

Certain writers of recent time have thrown dust into the eyes of teachers by endeavoring to set forth as a fact that Plato and other great thinkers of Greece denied the disciplinary value of mathematics in the general education of youth. These distortions have resulted from a paucity of quotation, combined with a failure to explain the technical uses of certain words quoted.

In his "Republic," Plato explained the education which citizens of his ideal State should receive. When he speaks of "arithmetic'" and "geometry," he uses terms which had the same meaning then as they have now. But "mathematics" and "reason" conveyed different meanings then—the former receiving a broader, the latter a much narrower interpretation. With Plato "mathematics" included not only arithmetic and geometry, but also astronomy and probably other physical sciences; "Reason" was used in a very specialized sense. He divides intellectual knowledge into two subgroups ("Republic," VI., 510B, 511): "hypothetical knowledge" and knowledge of

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[1] This is a special consideration under the general problem of the transfer of training. See H. O. Rugg, *Experimental Determination of Mental Discipline in School Studies,* Baltimore, 1916. J. W. A. Young, "Concerning Experiments to test the transfer of training" in *School Science and Mathematics,* Vol. 18, 1918, pp. 1–10.

"principles." He postulated the existence of two corresponding "faculties in the soul"—"understanding" and "reason." "Understanding" grapples with "hypothetical knowledge," "reason" with knowledge of "principles." To "reason" was to engage in dialectic (see Century Dictionary) *i.e.,* to draw *probable* conclusions, to determine the value of opinions, as opposed to the process of drawing *necessary* conclusions. The content of dialectic was largely ethics—"the good." Plato properly called mathematics "hypothetical knowledge." He taught that in the study of mathematics we exercise the "understanding," not the "reason"; we draw conclusions that are, as a rule, *necessary,* not *probable.* Plato ("Republic," VII., 531C) represents Socrates and Glaucon as engaged in the following dialogue:

"Now, when all these studies [arithmetic, geometry, astronomy] reach the point of intercommunion and connection with one another, and come to be considered in their mutual affinities, then, I think, but not till then, will the pursuit of them have a value for our objects [dialectic]; otherwise there is no profit in them. I suspect so; but you are speaking, Socrates, of a vast work. What do you mean? I said; the prelude of what? Do you not know that all this is but the prelude to the actual strain which we have to learn? For you surely would not regard the skilled mathematician as a dialectician? Assuredly not, he said; *I have hardly ever known a mathematician who was capable of reasoning.*"

What does Plato mean by this passage? Simply this, that a mere mathematician is not skilful in drawing *probable* conclusions, in the formation of opinions in metaphysical argumentation. He does not say that mathematical study is a hindrance to dialectic; quite the contrary; it is the opening door to dialectic. Mathematics is the "prelude," it is "but the prelude to the actual strain which we have to learn."

We proceed to give quotations from the dialogue in Plato's "Republic" which leave no doubt as to the rôle that arithmetic and geometry play in the training of citizens in his ideal State. In "Republic," VII., 522B, he says:

"We shall have to take something which is not special, but of universal application. What may that be? A something which all arts and sciences and intelligences use in common, and which every one first has to learn among the elements of education. What is that? The little matter of distinguishing one, two, and three—in a word, number and calculation:—do not all arts and sciences necessarily partake of them? Yes. Then the art of war partakes of them? To be sure. . . . Can we deny that a warrior should have a knowledge of arithmetic? Certainly he should, if he is to have the smallest understanding of military tactics, or indeed, I should rather say, if he is to be a man at all. . . . (VII., 525B): The man of war must learn the art of number or he will not know how to array his troops, and the philosopher also, because he has to rise out of the sea of change and lay hold of true being, and therefore he must be an arithmetician. . . . Then this is a kind of knowledge which legislation may fitly prescribe; and we must endeavour to persuade those who are to be the principal men of our State to go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only; nor again, like merchants or retail-traders, with a view to buying or selling, but for the sake of their military use, and of the soul herself. . . . I must add how charming the science is! and in how many ways it conduces to our desired end, if pursued in the spirit of a philosopher, and not of a shopkeeper! . . . (VII., 526B): And have you further observed, that *those who have a natural talent for calculation are generally quick at every other kind of knowledge; and even the dull, if they have had an arithmetical training, although they may derive no other advantage from it, always become much quicker than they would otherwise have been. . . .*"

"We are concerned with that part of geometry which relates to war; for in pitching a camp, or taking up a position, or closing or extending the lines of an army, or any other military manoeuvre, whether in actual battle or on a march, it will make all the difference whether a general is or is not a geometrician. Yes, I said, but for that purpose a very little of either geometry or calculation will be enough; the question relates rather to the greater and more advanced part of geometry—

whether that tends in any degree to make more easy the vision of the idea of good; and thither, as I was saying, all things tend which compel the soul to turn her gaze towards that place, where is the full perfection of being, which she ought, by all means, to behold. . . . (VII., 527C): Nothing should be more sternly laid down than that inhabitants of your fair city should by all means learn geometry. Moreover the science has indirect effects which are not small. . . . In all departments of knowledge, as experience proves, any one who has studied geometry is infinitely quicker of apprehension than one who has not. . . ."

That mathematical subjects are a necessary preliminary to the higher kinds of knowledge is brought out still more sharply in the folllowing passage ("Republic," VII., 536D):

"Calculation and geometry and all the other elements of instruction, which are a preparation for dialectic, should be presented to the mind in chidhood."

In another book, his "Laws" (V., 747), Plato sets forth the value of arithmetic in producing mental keenness, thus:

"The legislator is . . . to bid the citizens, as far as possible, not to lose sight of numerical order; for no single instrument of youthful education has such mighty power, both as regards domestic economy and politics, and in the arts, as the study of arithmetic. Above all, arithmetic stirs him up who is by nature sleepy and dull, and makes him quick to learn. . . ."

To sum up, then, Plato favored the study of arithmetic and geometry: (1) as not only necessary in war, but of universal application—hence to be prescribed by legislation; (2) as having great mind-training value, not for the pursuit of more advanced mathematics alone, but for the highest kind of knowledge, for dialectics—persons talented in calculation and geometry being "generally quick at every other kind of knowledge and even the dull . . . always become much quicker than they would otherwise have been"; (3) for the sake of "the soul herself."

We are informed by Tzetzes, a twelfth-century grammarian and poet of Constantinople, that Plato had an inscription over the entrance to his Academy at Athens: "Let no one who is unacquainted with geometry enter here." The authenticity of this statement has been attacked on the ground that neither Plato nor his contemporaries and immediate successors mention this inscription. Diogenes Laertius (about 200 A. D.) tells a similar story of Xenocrates, a successor of Plato as teacher in the Academy, to the effect that he declined to admit a pupil without mathematical training by saying, "Depart, for thou hast not the grip of philosophy."

According to Plutarch, Plato held that God was a great geometer, that for that reason Plato made a knowledge of geometry an indispensable prerequisite for the study of philosophy.

The statements of Tzetzes, Diogenes Laertius and Plutarch converge to the same central idea. Moreover, if we remember that quantities of Greek manuscripts have been lost, it seems quite probable that the alleged inscription is authentic. If it is apocryphal and must be placed with the story of William Tell or the premature fall of the Washington cherry tree, it nevertheless conveys a thought which is in complete harmony with Plato's teaching.

Aristotle has left no direct, explicit dictum on the educational value of arithmetic and geometry, even though his writings teem with reasoning mathematical in character. Perhaps the nearest approach is his statement that mathematics makes manifest *order* and *symmetry*. Aristotle cultivated mathematics without expressing himself on the disciplinary value any more than did the great professional Greek mathematicians, Euclid, Archimides, Apollonius of Perga and Diophantus.

Socrates is reported by Xenophon, in his "Memorabilia" (Book IV., Ch. 7), as strongly favoring the study of mathematics in so far as it is needed and useful in practical life. Socrates did not touch the question whether mathematics does or does not strengthen the rational powers generally. His center of interest lay in the fields of ethics and religion.

There were two sects among the ancients—the Pyrrhonics and the Epicureans—that were enemies of mathematics. The Pyrrhonics were skeptics and opposed all knowledge. Epicurus rejected the mathematical sciences because his opinions regarding nature were opposed to what was taught by mathematics.

It is worthy of notice that neither sect attacked mathematics on the ground that it did not train the mind. Apparently this question did not come before them for consideration.

Looking, therefore, upon Greek antiquity as a whole, we see that philosophers of great eminence, like Plato and some of his disciples, placed extraordinary emphasis upon the mind-training value of mathematics as a preparation for dialectical studies, and that *none* of the Greeks denied that value.

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